PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/upri20

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Published online: 10 May 2013.

To cite this article: Jean McGivney-Burelle & Fei Xue (2013) Flipping Calculus, PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies, 23:5, 477-486, DOI: 10.1080/10511970.2012.757571

To link to this article: http://dx.doi.org/10.1080/10511970.2012.757571

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Flipping Calculus

Jean McGivney-Burelle and Fei Xue

Abstract: In this paper we discuss flipping pedagogy and how it can transform the teaching and learning of calculus by applying pedagogical practices that are steeped in our understanding of how students learn most effectively. In particular, we describe the results of an exploratory study we conducted to examine the benefits and challenges of flipping a unit of study, the applications of the definite integral, in a Calculus II course. Data on student performance in flipped and non-flipped sections of the course are presented. In addition, students’ perceptions of the flipped unit are presented and discussed.

Keywords: Flipping, calculus, video, effective learning, inverted instruction.

1. INTRODUCTION

While the expression “flipping a course” is relatively new, this pedagogical strategy has been around for a number of years and is similar to “just-in-time teaching” [10] and “inverted instruction” [9]. Some tenets that underlie this type of pedagogy are that: (i) out-of-class time should be highly structured to best prepare students for in-class activities; (ii) it is useful to evaluate students’ pre-class preparation and for instructors to have access to this information; (iii) class time is better spent having students engage in cooperative problem solving and discussions rather than listening and taking notes; and, (iv) students benefit from more frequent structured practice and feedback in the classroom from a knowledgeable teacher.

There are numerous and varied models of flipped classrooms at the college level [e.g., 4, 5, 7, 11–14]; however, several general principles guide most instructors’ approach to this type of pedagogy. Generally speaking, students in a flipped course spend time out of class working individually to acquire...
basic knowledge of the course material— that is, definitions, facts, skills, and routine procedures which would traditionally be transmitted through a lecture. This knowledge acquisition is achieved through reading instructor-provided materials, watching videos or other online resources, and completing brief assessments before attending class or at the start of class. The majority of time in a flipped class is devoted to affording students an opportunity to assimilate their knowledge through structured group problem-solving activities, small group discussions, and whole-class discussions all under the guidance of a more knowledgeable instructor. In this model, the instructor circulates around the room, interacting with individuals or small groups, posing and answering questions, and providing clarification to the whole class when needed. This is a model of pedagogy that leverages the expertise of the instructor in providing feedback for students engaged in solving problems to assist them in integrating and applying their knowledge [13].

2. WHY FLIPPING MAKES SENSE

Our basic assumption is that students require less support when engaged in learning definitions, basic facts, skills, and routine procedures and require more instructor feedback when they are engaged in activities that involve higher-level reasoning. In a typical lecture emphasis is placed on the lower level of the cognitive domain of Bloom’s Taxonomy, including knowledge and comprehension with some emphasis on application. However, when students leave a lecture and attempt homework problems on their own they are often expected to engage in higher-level skills of analysis, synthesis, and evaluation without the support of their peers and the careful guidance and feedback from a teacher. In a flipped course, there are more opportunities for students to receive peer and instructor support while they are engaged in tasks that require more complex thinking and reasoning skills.

The model of deliberate practice proposed by Ericsson et al. [6] lends support for flipping pedagogy. In particular, they posit that one must engage in purposeful practice with an explicit goal of constant improvement in order to achieve expert performance. Applied to the context of teaching and learning mathematics, in order for students to develop the habits of mathematicians they must engage in highly structured activities explicitly directed at improving their mathematical understanding and performance. Furthermore, according to [6], student performance must be carefully monitored by instructors who can then provide cues for ways of improving performance. While we do not expect our students to become experts in the relatively short amount of time we spend with them in a typical calculus course, it is true that students in a flipped classroom have significantly more opportunities to engage in purposeful practice and receive immediate feedback from instructors than in a more traditional course.

Deslauriers et al. [5] conducted a well-publicized study on the use of deliberate practice in a large lecture physics course. The study compared
the amounts of learning achieved in an introductory physics course using two different instructional approaches—3 hours of traditional lectures given by an experienced highly rated instructor compared with 3 hours of instruction given by a trained but inexperienced instructor implementing pedagogy based on research in cognitive psychology and physics education. Specifically, this pedagogy included no formal lecturing, a cycle of pre-class readings and assessments, in class group tasks, discussions and clicker questions, and instructor feedback. Comparisons were made between two large sections ($N = 267$ and $N = 271$) of the physics course and indicated increased student attendance, higher engagement, and improved learning in the section taught using research-based instruction. The focus of this section of the course was:

> to have the students spend all their time in class engaged in deliberate practice at ‘thinking scientifically’ in the form of making and testing predictions and arguments about the relevant topics, solving problems, and critiquing their own reasoning and that of others [5].

A review of the literature revealed numerous papers describing the structure and benefits of flipping college physics, engineering, and technology courses (e.g., [4, 5, 7]). For example, Demetry [4] found that one benefit of flipping her engineering course was that students spent more time outside of class engaged in learning class content. Frydenberg [7] in his information technology course found that flipping pedagogy captured students’ interest, challenged students, and contributed to their learning. However, in mathematics, the study performed by Strayer [11, 12] was the only one identified that sought to examine flipping pedagogy in a college-level mathematics course. Specifically, Strayer [11, 12] compared the learning environments of an inverted and traditional introductory statistics class and found that while students in the flipped class were less satisfied with how they were oriented to the learning tasks in the course, they became more open to cooperative learning and innovative teaching methods. Given the growing interest in flipping pedagogy and the dearth of research studies in the context of college mathematics courses this is fertile ground for research.

### 3. THE IMPORTANCE OF CALCULUS

The need to improve the participation and performance of U.S. students in science, technology, and medicine (STEM) is widely acknowledged. At the college level, calculus is a course that has significant potential to serve as a pump for increasing the number students majoring in STEM fields. Furthermore, calculus is a foundation course for all STEM majors and, if taught well, should provide students with a positive and successful first-year experience and gateway into more advanced courses.
While there have been significant efforts over the past three decades to reform the teaching and learning of college-level calculus, this course remains a filter, rather than a pump, for science, technology, engineering, or mathematics majors. The 2010 College Board reported that of the students who entered college in 1995–1996 with the intention of pursuing a STEM major only 27% had succeeded by 2001, with calculus courses being one of the significant contributors to this loss. For example, during the Fall 2010 semester, 325,000 students were enrolled in a college or university Calculus I course, with 75% of them intending to major in science, engineering, or mathematics. However, while these students demonstrated above average mathematical aptitude (i.e., average SAT Math score of 652 with three-quarters of these students earning at least a 610) 27% received a D or F or withdrew from the course and a further 23% received a C [2].

Despite the growing body of evidence which suggests other instructional approaches are more effective in improving student engagement, satisfaction, and performance (e.g., [3, 8]), it is still the case that most college mathematics instructors believe that students learn best via lectures [1]. Concerns about the need to “cover the content” may drive mathematics faculty towards a lecture model of teaching since it is seen as the most efficient model for transmitting essential course information. The sense is that while it is understood that engaged learning pedagogies may be ideal, they take too much time and resources to implement. A basic assumption of our efforts to flip calculus is that while we are implementing instructional approaches that improve student engagement during class time we are also providing more structure and avenues for learning outside of class time so that students can uncover and make sense of the mathematics while we are still covering course content.

4. FLIPPING CALCULUS AT OUR UNIVERSITY

University of Hartford is a mid-sized private university in the Northeast. We offer multiple sections of calculus every semester. Approximately 25–30 students are enrolled in each. At our university the goal is to provide students with a calculus experience that will transform them from traditional learners into independent and intellectual members of the community. To this end, we continually refine our teaching and curriculum to improve student engagement, retention, and success, and seek to attract and retain more students to STEM majors. Presently, we use Stewart’s Calculus: Concepts and Contexts (fourth edition) text for our calculus sequence. The TI-89 graphing calculator and WeBWorK—an online homework system—are required in all sections of these courses. In addition, faculty members use Maple Worksheets, mathlets, videos, clickers, and e-textbooks to supplement their instruction.

During the Spring of 2012, the co-author of this paper was teaching two back-to-back sections of Calculus II and flipped the unit on Applications of
Integration (Chapter 6 in the Stewart text, including area, volume, arc length, average value of a function, and applications to physics and probability), in one of the two sections. This unit of study is a challenging one for students so we were particularly motivated to find ways to improve students’ learning in this unit. The structure of the flipping worked as follows. The instructor taught section A of Calculus II in a traditional manner, lecturing for most of every class period, for the entire semester. He also taught section B in a traditional manner with the exception of the one flipped unit in Chapter 6. During the flipped unit students in section B were expected to watch several short videos prior to every class meeting. For every section covered in Chapter 6 the instructor created and posted two to three short online videos (totaling approximately 15 minutes in length) of key lesson concepts and worked solutions to sample problems. At the start of every class of the flipped unit students in section B had to complete a one to two question entrance quiz to assess their out-of-class preparation. These quizzes took about 5 to 10 minutes and typically involved asking the students to write down a formula introduced in the videos or to solve the exact same problem demonstrated in the videos. The entrance quizzes were graded and accounted for about 2% of the final grade. During each class in the flipped unit the students in section B spent virtually the entire period working in small groups on problem sets ranging from five to seven problems in length. While the students worked the problems the instructor circulated around the room answering students’ questions and providing them feedback on their progress. In contrast, students in section A did not watch any assigned videos outside of class and spent most of their class time listening to the instructor’s lecture and taking notes. Throughout the semester students in both sections of the course were expected to complete WeBWorK homework problems for every section covered in the textbook. It should be noted that no students dropped either section of the course during the data collection period.

Student performance from sections A and B of the course on exams 1 and 2 were collected and are organized in Table 1 and Table 2. Note that in both sections students were presented the material in a traditional format for the first third of the courses—that is from the first day of the semester up until the first exam. Students in section B experienced flipped teaching from after exam 1 until exam 2. As shown in Table 1 and Table 2 the average exam score of students in sections A and B was approximately the same for the first exam. In contrast, on exam 2 the mean score of students in section B was five points higher and the median score was seven points higher than similar measures for section A.

In addition to exam scores, students’ grades on the WeBWorK assignments were also compared. As shown in Table 3, the average WeBWorK score for students in sections A and B on Chapter 5 was essentially the same. However, the average score for students in section B for the flipped unit (i.e., Chapter 6) was four points higher than the average score for students in section A.
Table 1. Section A Exams (non-flipped class, 29 students) (in percent)

<table>
<thead>
<tr>
<th></th>
<th>Exam 1 (Traditional)</th>
<th>Exam 2 (Traditional)</th>
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</thead>
<tbody>
<tr>
<td>Average</td>
<td>76.24</td>
<td>71.27</td>
</tr>
<tr>
<td>Median</td>
<td>82</td>
<td>73</td>
</tr>
</tbody>
</table>

Table 2. Section B Exams (flipped class, 31 students) (in percent)

<table>
<thead>
<tr>
<th></th>
<th>Exam 1 (Traditional)</th>
<th>Exam 2 (Flipped)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>77.48</td>
<td>76.48</td>
</tr>
<tr>
<td>Median</td>
<td>83</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 3. WebWork Grades (in percent) Comparison

<table>
<thead>
<tr>
<th></th>
<th>Section A</th>
<th>Section B</th>
</tr>
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<tbody>
<tr>
<td>Chapter 5 (non-flipped)</td>
<td>93.32</td>
<td>92.45</td>
</tr>
<tr>
<td>Chapter 6 (section A non-flipped, section B flipped)</td>
<td>77.45</td>
<td>81.48</td>
</tr>
</tbody>
</table>

One concern this instructor had was whether, during the flipped unit, students in section B would watch the videos. At the beginning of each class students indicated the amount of time they spent watching videos outside of class by anonymously registering their answers via clickers. Survey results indicated that 77.9% of these students watched the videos before every class and, on average, these students spent 21 minutes before every class watching or reviewing videos.

In addition to test scores, all of the students in section B completed a survey that attempted to capture their perceptions about the flipped unit. On this survey students were asked to rate the extent to which the videos helped them learn the material on a scale of 0 (not at all) to a 5 (a lot). The average response was 4.06. Furthermore, when asked, “What do you like most about the videos?” students most often said that:

- The videos featured the instructor and were short and easy to follow.
- They liked being able to pause and re-watch the videos at any time.
- They appreciated having videos of worked examples.
- The videos prompted students to see the big ideas of the section before class.
- They appreciated being able to take notes from the videos at their own pace.
- The videos were useful in reviewing before an exam.

Students in the flipped unit also appreciated the way in which class time was used during the flipped unit of study. Specifically, they said they liked being
able to do more challenging examples in class rather than listen to a lecture, they appreciated being able to work at their own pace in class, and they enjoyed having the instructor available in class to help them while they worked on problems.

In order to probe student perceptions more deeply another faculty member conducted a semi-structured focus group interview with four students in section B who volunteered to participate in the interview. Every student in the focus group indicated a strong preference for the flipping pedagogy over traditional instruction and they most appreciated having the videos to watch outside of class and how class time was spent. Some of their thoughts about flipping pedagogy are provided in the form of the following direct quotes:

The videos are a stress-free visual method of learning. . . . When he does it in class . . . you are worried about like getting the visual picture of it and you are worried about ‘do I understand this?’ ‘is this confusing’?

When you are in class . . . you can shake your head and you know every step . . . and then when you go back by yourself it is always a different story because there you have no guide to help you. Watching the videos and doing the examples with it . . . it was more of a self-tester . . . it’s like . . . ‘oh yeah, I understand this concept’. . . . To do it problems your own pace . . . was a lot more effective.

One student felt strongly about the benefits of how class time was spent during the flipped unit:

But how class time is used is the best thing about it . . . so it is nice to have Prof. X right there to say ‘hey this is how I figured it out’ does this work . . . and working with groups we can bounce ideas back and forth.

Another student said that the way class time was structured during the flipped unit made her feel more comfortable to ask questions:

I feel more free to ask questions too . . . because there is less of a time constraint. I would not ask questions when he was just lecturing because I would say ‘Okay, I don’t understand what just happened there but I am sure it will be explained later. I don’t want to hold up the whole class and rewind the last 5-10 minutes.’

While the size of the focus group was small there was evidence that these students strongly preferred the flipped unit to the non-flipped units of study.

We also asked the students to tell us what they did not like about the flipping pedagogy. While the majority of students preferred flipping to lectures
some students did not like that they could not ask the instructor questions while watching the videos. Consequently, when they did not understand the material in videos they struggled with the entrance quiz and the in-class group problems. Also, some students mentioned that if they forgot to watch the videos before class their time in class was less well spent. These students recommended that in a flipped course it would be helpful for the instructor to spend a few minutes at the start of class answering student questions about the video. Another suggestion was that after the entrance quiz the instructor should let the students who forgot to watch the video spend a few minutes watching the videos while the rest of the class got started on the in-class problem set.

5. LESSONS LEARNED

As a result of our instructional experiment we learned the following lessons about flipping pedagogy.

1. It is possible to cover the same amount of material in the same amount of time in a flipped unit of study as it through a traditional lecture.
2. There is some evidence that students learning via a flipping pedagogy outperform students who are in a traditional lecture.
3. Students are very satisfied with the simple videos/screencasts that we created. They appreciated hearing their instructor’s voice on the videos and liked watching the lesson and problem solutions develop line-by-line on the screen.
4. When the virtual TI-89 was demonstrated in the screencasts it allowed students to become more proficient in using the calculator.
5. Creating, editing, and posting videos are time-consuming endeavors as is the development of the in-class quizzes and problem sets. On average, for every class meeting, it took us about 1.5 hours to make one short video and an additional 45 minutes to prepare the quiz and in-class problem set. In contrast, preparing a traditional lecture typically takes us less time. However, once the initial technical and logistical problems were resolved, we were able to spend less time creating a video. Once a polished set of videos and course materials are created the preparation time will be significantly reduced.
6. Those new to flipping should expect many technology glitches especially when creating the first few videos. There are many choices to be made about the hardware (e.g., tablet PC, iPad, Slate PC, Smartpen) and software (e.g., Camtasia, Doceri, etc.) and much of it depends on what the instructor wants to appear in the video. There are also choices to be made about what type of video file to create (Mp4 in our case) and where to post the videos (e.g., Blackboard, YouTube, etc.).
7. Instead of creating one’s own videos, there are numerous existing videos that are accessible online. For example, Khan Academy has over 200 videos for the whole calculus series. In addition, many publishers have developed nice recorded videos for their textbooks.

Overall, there was enough evidence to indicate that our flipping pedagogy in calculus was effective and worth the significant investment of faculty time and effort. Students in the flipped section of the course preferred this type of pedagogy, particularly the availability of videos and the use of class time to solve problems, and fared better on homework and tests. We look forward to developing more flipped units for our calculus courses and to conducting further research on the impact of this type of pedagogy on student perceptions of and performance in mathematics. Furthermore, we believe flipping pedagogy has the potential to work well across a range of mathematics, as well as other STEM courses, and are encouraged by the growing national interest in this type of pedagogy which will certainly lead to new insights, strategies, and tools.

Note: Our sample materials for the flipped version of Arc Length, including links to videos on YouTube, entrance quiz, and in class problems set can be found in the link below: https://docs.google.com/open?id=0ByjVTvbymsmnWd5allvRzZhdzQ. Readers are invited to e-mail the authors for the videos and additional materials of other sections.

REFERENCES


BIOGRAPHICAL SKETCHES

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